Matrices

ISC Previous Years Board Questions

with Answers

2000 to Semester1-2021

1. Using matrices, solve the following system of equations: (ISC 2000)

$$x_1 - 2x_2 + 3x_3 = 4$$
$$2x_1 + x_2 - 3x_3 = 5$$
$$-x_1 + x_2 + 2x_3 = 3$$

2. Using matrix method, solve the following system of equations : (ISC 2001)

$$x - 2y + 3z = 6$$
$$x + 4y + z = 12$$
$$x - 3y + 2z = 1$$

3. Find the inverse of the matrix $A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$ (ISC 2002)

4. If $\begin{bmatrix} x^2 & 3 & 4 \\ 1 & 9 & 8 \end{bmatrix} + \begin{bmatrix} -3x & 1 & -5 \\ -3 & -2 & -6 \end{bmatrix} = \begin{bmatrix} 4 & 4 & -1 \\ -2 & 7 & 2 \end{bmatrix}$, find the value of x. (ISC 2003)

5. Using matrices, solve the following system of equations : (ISC 2003)

$$x + 2y = 5$$
$$y + 2z = 8$$
$$2x + z = 5$$

6. If the matrix $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, show that $A^{-1} = \frac{1}{19}A$. (ISC 2004)

7. Given that $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ and $A\,(adj\ A) = k \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, (ISC 2005)
find the value of k.

8. Given the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ -2 & 1 & 0 \\ 0 & -1 & 2 \end{bmatrix}$, compute A^{-1}. (ISC 2005)

9. Solve for x and y: $\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} + 2\begin{bmatrix} 2x \\ 3y \end{bmatrix} = 3\begin{bmatrix} 7 \\ -3 \end{bmatrix}$. (ISC 2006)

10. Find the adjoint of the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$ and hence find the matrix A^{-1}.

(ISC 2006)

11. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, show that $A^2 - 3I = 2A$. (ISC 2007)

12. Solve the following system of equations using matrices : (ISC 2007)

$$x + y + z = 6$$
$$x - y + z = 2$$
$$2x + y - z = 1$$

13. Find x and y, if $x + y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $x - y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$. (ISC 2008)

14. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, find A^{-1} and hence solve the following system of linear equations :

$$x + 2y - 3z = -4$$
$$2x + 3y + 2z = 2$$
$$3x - 3y - 4z = 11$$

(ISC 2008)

15. If $M(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, show that $M(x)M(y) = M(x + y)$. (ISC 2009)

16. Solve the following linear equations using the matrix method: (ISC 2009)

$$x + y + z = 9$$
$$2x + 5y + 7z = 52$$
$$2x + y - z = 0$$

17. If the matrix $A = \begin{bmatrix} 6 & x & 2 \\ 2 & -1 & 2 \\ -10 & 5 & 2 \end{bmatrix}$ is a singular matrix, find the value of x. (ISC 2010)

18. Using matrix method , solve the following equations:

$$5x + 3y + z = 16$$
$$2x + y + 3z = 19$$
$$x + 2y + 4z = 25$$

(ISC 2010)

19. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, find x such that $A^2 = x\,A - 2I$. Hence find A^{-1}. (ISC 2011)

20. Solve the following system of equations using matrix method : (ISC 2011)

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$
$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$
$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

21. Solve for x and y if $\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} + 2\begin{bmatrix} 2x \\ 3y \end{bmatrix} = 3\begin{bmatrix} 7 \\ -3 \end{bmatrix}$. (ISC 2012)

22. Find the product of the matrices A and B where : (ISC 2012)

$$A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}.$$

Hence, solve the following equations by matrix method:

$$x + y + 2z = 1$$
$$3x + 2y + z = 7$$
$$2x + y + 3z = 2$$

23. If $(B - 2I)(B - 3I) = 0$, where $B = \begin{bmatrix} 4 & 2 \\ -1 & y \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find the value of y.

(ISC 2013)

24. Find A^{-1}, where $A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & -2 \end{bmatrix}$. (ISC 2013)

Hence, solve the following system of linear equations:

$$4x + 2y + 3z = 2$$
$$x + y + z = 1$$
$$3x + y - 2z = 5$$

25. If $B = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find the values of x and y such that $B^2 + xI_2 = yB$. (ISC 2014)

26. Using matrix method, solve the following system of equation: (ISC 2014)

$$x - 2y = 10$$
$$2x + y + 3z = 8$$
$$-2y + z = 7$$

27. Find the value of k if $M = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $M^2 - k\,M - I_2 = 0$. (ISC 2015)

28. Given two matrices A and B (ISC 2015)

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 4 & 1 \\ 1 & -3 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 11 & -5 & -14 \\ -1 & -1 & 2 \\ -7 & 1 & 6 \end{bmatrix},$$

find AB and use this result to solve the following system of equations:

$$x - 2y + 3z = 6,\ x + 4y + z = 12,\ x - 3y + 2z = 1.$$

29. Find the matrix X for which : (ISC 2016)

$$\begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$$

30. Solve the following system of linear equations using matrix method: (ISC 2016)

$$3x + y + z = 1,\ 2x + 2z = 0,\ 5x + y + 2z = 2.$$

31. If the matrix $\begin{pmatrix} 6 & -x^2 \\ 2x - 15 & 10 \end{pmatrix}$ is symmetric, find the value of x. (ISC 2017)

32. Given $A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{pmatrix}$, find AB. (ISC 2017)

Using this result, solve the following system of equations:

$$x - y = 3,\ 2x + 3y + 4z = 17 \text{ and } y + 2z = 7$$

33. If $A = \begin{pmatrix} 5 & a \\ b & 0 \end{pmatrix}$ and A is symmetric matrix , show that $a = b$. (ISC 2018)

34. Using matrices, solve the following system of equations: (ISC 2018)

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

35. Using elementary transformation, find the inverse of the matrix : (ISC 2018)

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

36. Show that $(A + A')$ is symmetric matrix , if $A = \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix}$. (ISC 2019)

37. Solve the following system of linear equations using matrix method: (ISC 2019)

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 9$$
$$\frac{2}{x} + \frac{5}{y} + \frac{7}{z} = 52$$
$$\frac{2}{x} + \frac{1}{y} - \frac{1}{z} = 0$$

38. If $\begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}\begin{pmatrix} 1 & -3 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} -4 & 6 \\ -9 & x \end{pmatrix}$, find x. (ISC 2020)

39. Solve the following system of linear equations using matrices: (ISC 2020)

$$x - 2y = 10,\ 2x - y - z = 8,\ -2y + z = 7.$$

40. Select and write the correct option: (ISC Sem 1-2021)

If $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$, then the matrix A^2 is equal to:

(a) $A^2 = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 16 \end{pmatrix}$

(b) $A^2 = \begin{pmatrix} 0 & 0 & 4 \\ 0 & 9 & 0 \\ 16 & 0 & 0 \end{pmatrix}$

(c) $A^2 = \begin{pmatrix} 4 & 0 & 16 \\ 0 & 0 & 0 \\ 0 & 9 & 0 \end{pmatrix}$

(d) $A^2 = \begin{pmatrix} 0 & 0 & 0 \\ 4 & 9 & 16 \\ 0 & 0 & 0 \end{pmatrix}$

Answer: _____

41. Select and write the correct option: (ISC Sem 1-2021)

If $2\begin{pmatrix} a & 9 \\ 6 & d \end{pmatrix} + 3\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} = 3\begin{pmatrix} 3 & 5 \\ 4 & 6 \end{pmatrix}$, then the values of a and d respectively are:

(a) 3, 6
(b) 3, 5
(c) 3, 9
(d) 3, 7

Answer: _____

42. Select and write the correct option: (ISC Sem 1-2021)

If the matrix $A = \begin{pmatrix} 1 & x & -1 \\ -1 & 3 & 2 \\ 2 & 1 & 1 \end{pmatrix}$ is singular, then the value of $'x'$ is

(a) $\frac{8}{5}$
(b) $\frac{-8}{5}$
(c) $\frac{5}{8}$
(d) 1

Answer: _____

43. Select and write the correct option: (ISC Sem 1-2021)

If $A = \begin{pmatrix} -1 & 1 \\ 2 & 3 \end{pmatrix}$, then $A\ (adj\ A)$ is equal to:

(a) $\begin{pmatrix} 4 & 0 \\ 10 & 24 \end{pmatrix}$
(b) $\begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix}$
(c) $\begin{pmatrix} 4 & 5 \\ 5 & 4 \end{pmatrix}$
(d) $\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$

Answer: _____

44. Select and write the correct option: (ISC Sem 1-2021)

A school wants to award its students for their achievement in Sports, Music, and Literature with a total cash prize of ₹6000.
Three times the prize money for Literature added to the prize money given for sports is equal to ₹11000.
The prize money given for sports and Literature together is equal to two times of the prize money given for music.
If x, y, and z represent the prize money given for Sports, Music, and Literature respectively, then

(i) The set of linear equations representing the above information will be:

(a) $x + y + z = 6000,\ x + 3z = 11000$ and $x - 2y + z = 0$

(b) $x + y + z = 6000,\ x + 3z = 11000$ and $x + y + 2z = 0$
(c) $x + y + z = 6000,\ 3x + z = 11000$ and $x + y - 2z = 0$
(d) $x + y + z = 6000,\ x + 3z = 11000$ and $2x + 2y - z = 0$

Answer: _____

(ii) Consider $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{pmatrix}$

Then the value of $|A|$ is

(a) 3
(b) 6
(c) 4
(d) -1

Answer: _____

(iii) The $adj\ A = \begin{pmatrix} 6 & p & 3 \\ q & 0 & r \\ -2 & 3 & -1 \end{pmatrix}$, the values of p, q and r respectively will be:

(a) $3,\ 2, -2$
(b) $3, -2, -2$
(c) $-3,\ 2,\ -2$
(d) $3,\ 2,\ 2$

Answer: _____

(iv) Using $|A|$ and $adj\ A$, calculate the prize money for sports (x).

(a) $x = 1500$
(b) $x = 500$
(c) $x = 1000$
(d) $x = 2000$

Answer: _____

Answer

1.	$x_1 = 4, x_2 = 3, x_3 = 2$	2.	$x = 1, y = 2, z = 3$
3.	$\begin{bmatrix} 1 & -1/2 \\ -2 & 3/2 \end{bmatrix}$	4.	$4, -1$
5.	$x = 1, y = 2, z = 3$	7.	$k = 1$
8.	$\frac{1}{6}\begin{bmatrix} 2 & -2 & -2 \\ 4 & 2 & -4 \\ 2 & 1 & 1 \end{bmatrix}$	9.	$x = 3, -7,\ y = -3, -3$
10.	$A^{-1} = \frac{1}{20}\begin{bmatrix} 2 & 6 & 4 \\ 21 & -7 & -8 \\ -18 & 6 & 4 \end{bmatrix}$	12.	$x = 1, y = 2, z = 3$

13.	$x = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix},\ y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$	14.	$A^{-1} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix},$ $x = 3,\ y = -2,\ z = 1$
16.	$x = 1,\ y = 3,\ z = 5$	17.	$x = -3$
18.	$x = 1,\ y = 2,\ z = 5.$	19.	$x = 1,\ A^{-1} = \frac{1}{2}\begin{bmatrix} -2 & 2 \\ -4 & 3 \end{bmatrix}$
20.	$x = 2, y = 3,\ z = 5$	21.	$x = -7,\ 3,\ y = -3,\ -3$
22.	$AB = 4\,I,\ x = 2,\ y = 1,\ z = -1$	23.	$y = 1$
24.	$A^{-1} = -\frac{1}{8}\begin{bmatrix} -3 & 7 & -1 \\ 5 & -17 & -1 \\ -2 & 2 & 2 \end{bmatrix},$ $x = \frac{1}{2}, y = \frac{3}{2}, z = -1$	25.	$x = 8,\ y = 8$
26.	$x = 4,\ y = -3,\ z = 1$	27.	$k = 4$
28.	$AB = -8\,I_3,\ x = 1,\ y = 2,\ z = 3$	29.	$\begin{bmatrix} -3 & -14 \\ 4 & 17 \end{bmatrix}$
30.	$x = 1,\ y = -1,\ z = -1$	31.	$-5,\ 3$
32.	$AB = 6\,I_3,\ x = 2,\ y = -1,\ z = 4$	34.	$x = 1,\ y = 2,\ z = 3$
35.	$\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$	36.	$A + A' = \begin{bmatrix} 4 & 7 \\ 7 & 10 \end{bmatrix}$
37.	$x = 1,\ y = \frac{1}{3},\ z = \frac{1}{5}$	38.	$x = 13$
39.	$x = 0,\ y = -5,\ z = -3$	40.	(a)
41.	(a)	42.	(b)
43.	(b)	44.	[i] (a) [ii] (b) [iii] (c) [iv] (b)

Matrices

Solutions of Previous Years ISC Board Questions

2000 to 2020

1. Using matrices, solve the following system of equations : (ISC 2000)

$$x_1 - 2x_2 + 3x_3 = 4$$
$$2x_1 + x_2 - 3x_3 = 5$$
$$-x_1 + x_2 + 2x_3 = 3$$

Solution : The given linear equations are

$$\begin{aligned} x_1 - 2x_2 + 3x_3 &= 4 \\ 2x_1 + x_2 - 3x_3 &= 5 \qquad \text{---------- (1)} \\ -x_1 + x_2 + 2x_3 &= 3 \end{aligned}$$

Let $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & -3 \\ -1 & 1 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 5 \\ 3 \end{bmatrix}$

Then the equation (1) can be written as $AX = B$

Now $|A| = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & -3 \\ -1 & 1 & 2 \end{vmatrix}$

$$= 1\begin{vmatrix} 1 & -3 \\ 1 & 2 \end{vmatrix} - (-2)\begin{vmatrix} 2 & -3 \\ -1 & 2 \end{vmatrix} + 3\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix}$$
$$= (2+3) + 2(4-3) + 3(2+1)$$
$$= 5 + 2 + 9 = 16 \neq 0$$

$$adj.A = \begin{bmatrix} +\begin{vmatrix} 1 & -3 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & -3 \\ -1 & 2 \end{vmatrix} & +\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} \\ -\begin{vmatrix} -2 & 3 \\ 1 & 2 \end{vmatrix} & +\begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & -2 \\ -1 & 1 \end{vmatrix} \\ +\begin{vmatrix} -2 & 3 \\ 1 & -3 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 2 & -3 \end{vmatrix} & +\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} (2+3) & -(4-3) & (2+1) \\ -(-4-3) & (2+3) & -(1-2) \\ (6-3) & -(-3-6) & (1+4) \end{bmatrix}^T$$

$$= \begin{bmatrix} 5 & -1 & 3 \\ 7 & 5 & 1 \\ 3 & 9 & 5 \end{bmatrix}^T$$

$$= \begin{bmatrix} 5 & 7 & 3 \\ -1 & 5 & 9 \\ 3 & 1 & 5 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}(adj.A) = \frac{1}{16}\begin{bmatrix} 5 & 7 & 3 \\ -1 & 5 & 9 \\ 3 & 1 & 5 \end{bmatrix}$$

$\because AX = B \quad \Rightarrow \quad X = A^{-1}B$

$$\Rightarrow \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \frac{1}{16}\begin{bmatrix} 5 & 7 & 3 \\ -1 & 5 & 9 \\ 3 & 1 & 5 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \\ 3 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \frac{1}{16}\begin{bmatrix} 20 + 35 + 9 \\ -4 + 25 + 27 \\ 12 + 5 + 15 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \frac{1}{16} \begin{bmatrix} 64 \\ 48 \\ 32 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$$

$\therefore \ x_1 = 4, \ x_2 = 3, \ x_3 = 2$ Answer

2. Using matrix method, solve the following system of equations: (ISC 2001)

$$x - 2y + 3z = 6, \ x + 4y + z = 12, \ x - 3y + 2z = 1 .$$

Solution : The given linear equations are

$$x - 2y + 3z = 6$$
$$x + 4y + z = 12 \quad \text{---------- (1)}$$
$$x - 3y + 2z = 1$$

Let $A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 4 & 1 \\ 1 & -3 & 2 \end{bmatrix}, \ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \ B = \begin{bmatrix} 6 \\ 12 \\ 1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & -2 & 3 \\ 1 & 4 & 1 \\ 1 & -3 & 2 \end{vmatrix} = \begin{vmatrix} 1 & -2 & 3 \\ 0 & 6 & -2 \\ 0 & -1 & -1 \end{vmatrix} = -6 - 2 = -8 \neq 0$$

$$adj.A = \begin{bmatrix} + \begin{vmatrix} 4 & 1 \\ -3 & 2 \end{vmatrix} & - \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} & + \begin{vmatrix} 1 & 4 \\ 1 & -3 \end{vmatrix} \\ - \begin{vmatrix} -2 & 3 \\ -3 & 2 \end{vmatrix} & + \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} & - \begin{vmatrix} 1 & -2 \\ 1 & -3 \end{vmatrix} \\ + \begin{vmatrix} -2 & 3 \\ 4 & 1 \end{vmatrix} & - \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} & + \begin{vmatrix} 1 & -2 \\ 1 & 4 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} (8+3) & -(2-1) & (-3-4) \\ -(-4+9) & (2-3) & -(-3+2) \\ (-2-12) & -(1-3) & (4+2) \end{bmatrix}^T$$

$$= \begin{bmatrix} 11 & -1 & -7 \\ -5 & -1 & 1 \\ -14 & 2 & 6 \end{bmatrix}^T$$

$$= \begin{bmatrix} 11 & -5 & -14 \\ -1 & -1 & 2 \\ -7 & 1 & 6 \end{bmatrix}$$

$$\therefore \ A^{-1} = \frac{1}{|A|} adj.A = \frac{1}{-8} \begin{bmatrix} 11 & -5 & -14 \\ -1 & -1 & 2 \\ -7 & 1 & 6 \end{bmatrix}$$

Now $AX = B$

$$\Rightarrow \quad X = A^{-1} B$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{8} \begin{bmatrix} 11 & -5 & -14 \\ -1 & -1 & 2 \\ -7 & 1 & 6 \end{bmatrix} \begin{bmatrix} 6 \\ 12 \\ 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{8} \begin{bmatrix} 66 - 60 - 14 \\ -6 - 12 + 2 \\ -42 + 12 + 6 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{8} \begin{bmatrix} -8 \\ -16 \\ -24 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\therefore \ x = 1, \ \ y = 2, \ z = 3$ Answer

3. Find the inverse of the matrix $A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$ (ISC 2002)

Solution : Given $A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$

$$\therefore |A| = 6 - 4 = 2$$

$$adj.A = \begin{bmatrix} 2 & -4 \\ -1 & 3 \end{bmatrix}^T$$

$$= \begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$$

$$\therefore \ A^{-1} = \frac{1}{2}(adj.A)$$

$$= \frac{1}{2}\begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$$ Answer

4. If $\begin{bmatrix} x^2 & 3 & 4 \\ 1 & 9 & 8 \end{bmatrix} + \begin{bmatrix} -3x & 1 & -5 \\ -3 & -2 & -6 \end{bmatrix} = \begin{bmatrix} 4 & 4 & -1 \\ -2 & 7 & 2 \end{bmatrix}$, find the value of x. (ISC 2003)

Solution : $\begin{bmatrix} x^2 & 3 & 4 \\ 1 & 9 & 8 \end{bmatrix} + \begin{bmatrix} -3x & 1 & -5 \\ -3 & -2 & -6 \end{bmatrix} = \begin{bmatrix} 4 & 4 & -1 \\ -2 & 7 & 2 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x^2 - 3x & 3+1 & 4-5 \\ 1-3 & 9-2 & 8-6 \end{bmatrix} = \begin{bmatrix} 4 & 4 & -1 \\ -2 & 7 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x^2 - 3x & 4 & -1 \\ -2 & 7 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 4 & -1 \\ -2 & 7 & 2 \end{bmatrix}$$

$$\therefore \quad x^2 - 3x = 4$$

$$\Rightarrow \quad x^2 - 3x - 4 = 0$$

$$\Rightarrow \quad x^2 - 4x + x - 4 = 0$$

$$\Rightarrow \quad x(x-4) + (x-4) = 0$$

$$\Rightarrow \quad (x-4)(x+1) = 0$$

$$\therefore \quad x = 4, -1$$ Answer

5. Using matrices, solve the following system of equations : (ISC 2003)

$$x + 2y = 5$$
$$y + 2z = 8$$
$$2x + z = 5$$

Solution : The given equation may be written as

$$x + 2y + 0 = 5$$
$$0 + y + 2z = 8 \quad \text{---------- (1)}$$
$$2x + 0 + z = 5$$

Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 5 \\ 8 \\ 5 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{vmatrix} = 1\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} - 2\begin{vmatrix} 0 & 2 \\ 2 & 1 \end{vmatrix} = 1 + 8 = 9 \neq 0$$

$$adj.A = \begin{bmatrix} +\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 0 & 2 \\ 2 & 1 \end{vmatrix} & +\begin{vmatrix} 0 & 1 \\ 2 & 0 \end{vmatrix} \\ -\begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} & +\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix} \\ +\begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} & +\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & 4 & -2 \\ -2 & 1 & 4 \\ 4 & -2 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -2 & 4 \\ 4 & 1 & -2 \\ -2 & 4 & 1 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}\,(adj.A) = \frac{1}{9}\begin{bmatrix} 1 & -2 & 4 \\ 4 & 1 & -2 \\ -2 & 4 & 1 \end{bmatrix}$$

Now $AX = B$

$$X = A^{-1}B$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 1 & -2 & 4 \\ 4 & 1 & -2 \\ -2 & 4 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 8 \\ 5 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 5 - 16 + 20 \\ 20 + 8 - 10 \\ -10 + 32 + 5 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 9 \\ 18 \\ 27 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\therefore \quad x = 1, \ y = 2, \ z = 3$ Answer

6. If the matrix $A = \begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix}$, show that $A^{-1} = \frac{1}{19}A$. (ISC 2004)

Solution : Given $A = \begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix}$

$$|A| = \begin{vmatrix} 2 & 3 \\ 5 & -2 \end{vmatrix} = -4 - 15 = -19 \neq 0$$

$$adj.A = \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{-19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$$

$$= \frac{1}{19}\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$$

$$= \frac{1}{19}A \qquad \text{Proved}$$

7. Given that $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ and $A(adj.A) = k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find the value of k. (ISC 2005)

Solution : Given $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$

$$\therefore \ |A| = cos^2x + sin^2x = 1 \text{ and } adj.A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$$

$$\therefore \ A(adj.A) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}\begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$$

$$= \begin{bmatrix} cos^2x + sin^2x & -\cos x \sin x + \sin x \cos x \\ -\sin x \cos x + \cos x \sin x & sin^2x + cos^2x \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Comparing $k = 1$ Answer

8. Given the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ -2 & 1 & 0 \\ 0 & -1 & 2 \end{bmatrix}$, compute A^{-1}. (ISC 2005)

Solution : $$|A| = \begin{vmatrix} 1 & 0 & 2 \\ -2 & 1 & 0 \\ 0 & -1 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 0 \\ -1 & 2 \end{vmatrix} + 2\begin{vmatrix} -2 & 1 \\ 0 & -1 \end{vmatrix} = 2 + 4 = 6 \neq 0$$

$$adj.A = \begin{bmatrix} +\begin{vmatrix} 1 & 0 \\ -1 & 2 \end{vmatrix} & -\begin{vmatrix} -2 & 0 \\ 0 & 2 \end{vmatrix} & +\begin{vmatrix} -2 & 1 \\ 0 & -1 \end{vmatrix} \\ -\begin{vmatrix} 0 & 2 \\ -1 & 2 \end{vmatrix} & +\begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} \\ +\begin{vmatrix} 0 & 2 \\ 1 & 0 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ -2 & 0 \end{vmatrix} & +\begin{vmatrix} 1 & 0 \\ -2 & 1 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} 2 & 4 & 2 \\ -2 & 2 & 1 \\ -2 & -4 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 2 & -2 & -2 \\ 4 & 2 & -4 \\ 2 & 1 & 1 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}(adj.A)$$

$$= \frac{1}{6}\begin{bmatrix} 2 & -2 & -2 \\ 4 & 2 & -4 \\ 2 & 1 & 1 \end{bmatrix} \qquad \text{Answer}$$

9. Solve for x and y : $\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} + 2\begin{bmatrix} 2x \\ 3y \end{bmatrix} = 3\begin{bmatrix} 7 \\ -3 \end{bmatrix}$. (ISC 2006)

Solution : Given $\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} + 2\begin{bmatrix} 2x \\ 3y \end{bmatrix} = 3\begin{bmatrix} 7 \\ -3 \end{bmatrix}$

$$\Rightarrow \quad \begin{bmatrix} x^2 + 4x \\ y^2 + 6y \end{bmatrix} = \begin{bmatrix} 21 \\ -9 \end{bmatrix}$$

$$\therefore \quad x^2 + 4x = 21 \quad \text{and} \quad y^2 + 6y = -9$$

$$\Rightarrow \quad x^2 + 4x - 21 = 0 \quad \text{and} \quad y^2 + 6y + 9 = 0$$

$$\Rightarrow \quad x^2 + 7x - 3x - 21 = 0 \quad \text{and} \quad y^2 + 3y + 3y + 9 = 0$$

$$\Rightarrow \quad (x+7)(x-3) = 0 \quad \text{and} \quad (y+3)(y+3) = 0$$

$$\therefore \quad x = -7,\ 3 \quad \text{and} \quad y = -3, -3 \qquad \text{Answer}$$

10. Find the adjoint of the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$ and hence find the matrix A^{-1}.

Solution : $|A| = \begin{vmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{vmatrix} = 1\begin{vmatrix} 4 & 5 \\ -6 & -7 \end{vmatrix} - 1\begin{vmatrix} 3 & 4 \\ 0 & -6 \end{vmatrix} = (-28 + 30) + 18$

$$= 20 \neq 0$$

$$adj.A = \begin{bmatrix} +\begin{vmatrix} 4 & 5 \\ -6 & -7 \end{vmatrix} & -\begin{vmatrix} 3 & 5 \\ 0 & -7 \end{vmatrix} & +\begin{vmatrix} 3 & 4 \\ 0 & -6 \end{vmatrix} \\ -\begin{vmatrix} 0 & -1 \\ -6 & -7 \end{vmatrix} & +\begin{vmatrix} 1 & -1 \\ 0 & -7 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 0 & -6 \end{vmatrix} \\ +\begin{vmatrix} 0 & -1 \\ 4 & 5 \end{vmatrix} & -\begin{vmatrix} 1 & -1 \\ 3 & 5 \end{vmatrix} & +\begin{vmatrix} 1 & 0 \\ 3 & 4 \end{vmatrix} \end{bmatrix}^T$$

$$\Rightarrow \quad adj.A = \begin{bmatrix} 2 & 21 & -18 \\ 6 & -7 & 6 \\ 4 & -8 & 4 \end{bmatrix}^T$$

$$\Rightarrow \quad adj.A = \begin{bmatrix} 2 & 6 & 4 \\ 21 & -7 & -8 \\ -18 & 6 & 4 \end{bmatrix} \qquad \text{Answer}$$

and $A^{-1} = \frac{1}{20}\begin{bmatrix} 2 & 6 & 4 \\ 21 & -7 & -8 \\ -18 & 6 & 4 \end{bmatrix}$ Answer

11. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, show that $A^2 - 3I = 2A$. (ISC 2007)

Solution : $A^2 = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1+4 & 2+2 \\ 2+2 & 4+1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$

$\therefore \quad A^2 - 3I = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} = 2\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = 2I$ Proved

12. Solve the following system of equations using matrices: (ISC 2007)

$$x + y + z = 6$$
$$x - y + z = 2$$
$$2x + y - z = 1$$

Solution : Given $\quad x + y + z = 6$
$x - y + z = 2$
$2x + y - z = 1$

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix} = \begin{vmatrix} 0 & 2 & 0 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix} = -2\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = 6 \neq 0$$

$$adj.A = \begin{bmatrix} +\begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} & +\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} & +\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} \\ +\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} & +\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 3 & 3 \\ 2 & -3 & 1 \\ 2 & 0 & -2 \end{bmatrix}^T = \begin{bmatrix} 0 & 2 & 2 \\ 3 & -3 & 0 \\ 3 & 1 & -2 \end{bmatrix}$$

$\therefore \quad A^{-1} = \frac{1}{|A|}(adj.A)$

$$= \frac{1}{6}\begin{bmatrix} 0 & 2 & 2 \\ 3 & -3 & 0 \\ 3 & 1 & -2 \end{bmatrix}$$

Now $\quad AX = B$

$\Rightarrow \quad X = A^{-1}B$

$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 0 & 2 & 2 \\ 3 & -3 & 0 \\ 3 & 1 & -2 \end{bmatrix}\begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 4+2 \\ 18-6 \\ 18+2-2 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 6 \\ 12 \\ 18 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$\therefore \quad x = 1, \quad y = 2, \quad z = 3$ Answer

13. Find x and y, if $x + y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $x - y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$. (ISC 2008)

Solution : $x + y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ --------- (1)

$x - y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ ---------- (2)

Adding (1) and (2), we get $2x = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$

$\therefore \quad x = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$

Substituting the value of x in (1), $\begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} + y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$

$\therefore \quad y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$ Answer

14. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, find A^{-1} and hence solve the following system of linear equations:

$x + 2y - 3z = -4$

$2x + 3y + 2z = 2$ (ISC 2008)

$3x - 3y - 4z = 11$

Solution : $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$

$\therefore \quad |A| = 1(-12 + 6) - 2(-8 - 6) - 3(-6 - 9) = -6 + 28 + 45 = 67 \neq 0$

$$\therefore \quad adj.A = \begin{bmatrix} +\begin{vmatrix} 3 & 2 \\ -3 & -4 \end{vmatrix} & -\begin{vmatrix} 2 & 2 \\ 3 & -4 \end{vmatrix} & +\begin{vmatrix} 2 & 3 \\ 3 & -3 \end{vmatrix} \\ -\begin{vmatrix} 2 & -3 \\ -3 & -4 \end{vmatrix} & +\begin{vmatrix} 1 & -3 \\ 3 & -4 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 3 & -3 \end{vmatrix} \\ +\begin{vmatrix} 2 & -3 \\ 3 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & -3 \\ 2 & 2 \end{vmatrix} & +\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} (-12 + 6) & -(-8 - 6) & (-6 - 9) \\ -(-8 - 9) & (-4 + 9) & -(-3 - 6) \\ (4 + 9) & -(2 + 6) & (3 - 4) \end{bmatrix}^T$$

$$= \begin{bmatrix} -6 & 14 & -15 \\ 17 & 5 & 9 \\ 13 & -8 & -1 \end{bmatrix}^T$$

$$= \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

$\therefore \quad A^{-1} = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$

Let $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \; B = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$

Now $AX = B$

$\therefore \quad X = A^{-1}B$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix} \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{67} \begin{bmatrix} 24 + 34 + 143 \\ -56 + 10 - 88 \\ 60 + 18 - 11 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{67}\begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

$\therefore \quad x = 3, \ y = -2, \ z = 1$ Answer

15. If $M(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, show that $M(x)M(y) = M(x+y)$. (ISC 2009)

Given $M(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$\therefore \quad M(x) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ and $M(y) = \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$

$$\therefore \quad M(x)M(y) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}\begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$$

$$= \begin{bmatrix} \cos x\cos y - \sin x\sin y & \cos x\sin y + \sin x\cos y \\ -\sin x\cos y - \cos x\sin y & -\sin x\sin y + \cos x\cos y \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix}$$

$= M(x+y)$ Proved

16. Solve the following linear equations using the matrix method : (ISC 2009)

$$x + y + z = 9$$
$$2x + 5y + 7z = 52$$
$$2x + y - z = 0$$

Solution : Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix}$

$$\therefore \quad |A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 3 & 5 \\ 2 & -1 & -3 \end{vmatrix} = -9 + 5 = -4 \neq 0$$

$$adj.A = \begin{bmatrix} +\begin{vmatrix} 5 & 7 \\ 1 & -1 \end{vmatrix} & -\begin{vmatrix} 2 & 7 \\ 2 & -1 \end{vmatrix} & +\begin{vmatrix} 2 & 5 \\ 2 & 1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} & +\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} \\ +\begin{vmatrix} 1 & 1 \\ 5 & 7 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 2 & 7 \end{vmatrix} & +\begin{vmatrix} 1 & 1 \\ 2 & 5 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} (-5-7) & -(-2-14) & (2-10) \\ -(-1-1) & (-1-2) & -(1-2) \\ (7-5) & -(7-2) & (5-2) \end{bmatrix}^T$$

$$= \begin{bmatrix} -12 & 16 & -8 \\ 2 & -3 & 1 \\ 2 & -5 & 3 \end{bmatrix}^T$$

$$= \begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{-4}\begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$$

Now $AX = B$

$X = A^{-1}B$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-4}\begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}\begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-4}\begin{bmatrix} -108 + 104 \\ 144 - 156 \\ -72 + 52 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-4}\begin{bmatrix} -4 \\ -12 \\ -20 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

$$\therefore \quad x = 1, \quad y = 3, \quad z = 5$$ Answer

17. If the matrix $A = \begin{bmatrix} 6 & x & 2 \\ 2 & -1 & 2 \\ -10 & 5 & 2 \end{bmatrix}$ is a singular matrix, find the value of x. (ISC 2010)

Solution : Given $A = \begin{bmatrix} 6 & x & 2 \\ 2 & -1 & 2 \\ -10 & 5 & 2 \end{bmatrix}$ is a singular matrix.

$$\therefore \quad |A| = \begin{vmatrix} 6 & x & 2 \\ 2 & -1 & 2 \\ -10 & 5 & 2 \end{vmatrix} = 0$$

$$\therefore \quad 6\begin{vmatrix} -1 & 2 \\ 5 & 2 \end{vmatrix} - x\begin{vmatrix} 2 & 2 \\ -10 & 2 \end{vmatrix} + 2\begin{vmatrix} 2 & -1 \\ -10 & 5 \end{vmatrix} = 0$$

$$\Rightarrow \quad 6(-2 - 10) - x(4 + 20) + 2(10 - 10) = 0$$

$$\Rightarrow \quad -72 - 24x = 0$$

$$\Rightarrow \quad x = -3$$ Answer

18. Using matrix method, solve the following equations : $5x + 3y + z = 16$, $2x + y + 3z = 19$, $x + 2y + 4z = 25$. (ISC 2010)

Solution : Given
$$5x + 3y + z = 16$$
$$2x + y + 3z = 19$$
$$x + 2y + 4z = 25$$

Let $A = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 16 \\ 19 \\ 25 \end{bmatrix}$

$$\therefore \quad |A| = \begin{vmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{vmatrix} = 5\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} - 3\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$$

$$= 5(4 - 6) - 3(8 - 3) + 1(4 - 1) = -10 - 15 + 3 = -22 \neq 0$$

$$adj.A = \begin{bmatrix} +\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} & -\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} & +\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} \\ -\begin{vmatrix} 3 & 1 \\ 2 & 4 \end{vmatrix} & +\begin{vmatrix} 5 & 1 \\ 1 & 4 \end{vmatrix} & -\begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} \\ +\begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} & -\begin{vmatrix} 5 & 1 \\ 2 & 3 \end{vmatrix} & +\begin{vmatrix} 5 & 3 \\ 2 & 1 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} (4 - 6) & -(8 - 3) & (4 - 1) \\ -(12 - 2) & (20 - 1) & -(10 - 3) \\ (9 - 1) & -(15 - 2) & (5 - 6) \end{bmatrix}^T$$

$$= \begin{bmatrix} -2 & -5 & 3 \\ -10 & 19 & -7 \\ 8 & -13 & -1 \end{bmatrix}^T$$

$$= \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{-22}\begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$$

Now $AX = B$

$\therefore \quad X = A^{-1}B$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-22}\begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}\begin{bmatrix} 16 \\ 19 \\ 25 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-22}\begin{bmatrix} -32 - 190 + 200 \\ -80 + 361 - 325 \\ 48 - 133 - 25 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-22}\begin{bmatrix} -22 \\ -44 \\ -110 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

$\therefore \quad x = 1, \ y = 2, \ z = 5$ Answer

19. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, find x such that $A^2 = xA - 2I$. Hence find A^{-1}. (ISC 2011)

Solution : $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

$$\therefore \quad A^2 = A.A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

Given $A^2 = xA - 2I$

$$\Rightarrow \quad \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = x\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3x & -2x \\ 4x & -2x \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3x-2 & -2x \\ 4x & -2x-2 \end{bmatrix}$$

$\Rightarrow \quad 4x = 4$

$\Rightarrow \quad x = 1$ Answer

$\therefore \quad A^2 = A - 2I$ ----------(1)

Pre-multiplying equation (1) by A^{-1} on both sides

$$(A^{-1}A)A = A^{-1}A - 2A^{-1}I$$

$\Rightarrow \quad IA = I - 2A^{-1} \quad [\because A^{-1}A = I \ and \ A^{-1}I = A^{-1}]$

$\Rightarrow \quad A = I - 2A^{-1} \quad [IA = A]$

$\Rightarrow \quad 2A^{-1} = I - A$

$$\Rightarrow \quad 2A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\Rightarrow \quad 2A^{-1} = \begin{bmatrix} -2 & +2 \\ -4 & 3 \end{bmatrix}$$

$$\Rightarrow \quad A^{-1} = \frac{1}{2}\begin{bmatrix} -2 & +2 \\ -4 & 3 \end{bmatrix}$$ Answer

20. Solve the following system of equations using matrix method : (ISC 2011)

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \quad \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1, \quad \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$

Solution : Let $\frac{1}{x} = u, \quad \frac{1}{y} = v, \quad \frac{1}{z} = w$

The given equation may be written as

$$2u + 3v + 10w = 4$$
$$4u - 6v + 5w = 1$$
$$6u + 9v - 20w = 2$$

Let $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$, $X = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$$\therefore \quad |A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}$$

$$= 2\begin{vmatrix} -6 & 5 \\ 9 & -20 \end{vmatrix} - 3\begin{vmatrix} 4 & 5 \\ 6 & -20 \end{vmatrix} + 10\begin{vmatrix} 4 & -6 \\ 6 & 9 \end{vmatrix}$$
$$= 2(120 - 45) - 3(-80 - 30) + 10(36 + 36)$$
$$= 2 \times 75 + 3 \times 110 + 10 \times 72$$
$$= 1200 \neq 0$$

$$adj\ A = \begin{bmatrix} +\begin{vmatrix} -6 & 5 \\ 9 & -20 \end{vmatrix} & -\begin{vmatrix} 4 & 5 \\ 6 & -20 \end{vmatrix} & +\begin{vmatrix} 4 & -6 \\ 6 & 9 \end{vmatrix} \\ -\begin{vmatrix} 3 & 10 \\ 9 & -20 \end{vmatrix} & +\begin{vmatrix} 2 & 10 \\ 6 & -20 \end{vmatrix} & -\begin{vmatrix} 2 & 3 \\ 6 & 9 \end{vmatrix} \\ +\begin{vmatrix} 3 & 10 \\ -6 & 5 \end{vmatrix} & -\begin{vmatrix} 2 & 10 \\ 4 & 5 \end{vmatrix} & +\begin{vmatrix} 2 & 3 \\ 4 & -6 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} (120 - 45) & -(-80 - 30) & (36 + 36) \\ -(-60 - 90) & (-40 - 60) & -(18 - 18) \\ (15 + 60) & -(10 - 40) & (-12 - 12) \end{bmatrix}^T$$

$$= \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}^T$$

$$= \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

Now $AX = B$

$$\Longrightarrow \quad X = A^{-1}B$$

$$\Longrightarrow \quad \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$\Longrightarrow \quad \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 300 + 150 + 150 \\ 440 - 100 + 60 \\ 288 + 0 - 48 \end{bmatrix}$$

$$\Longrightarrow \quad \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$$

$$\Longrightarrow \quad \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/3 \\ 1/5 \end{bmatrix}$$

$$\therefore \quad u = 1/2, \quad v = 1/3, \quad w = 1/5$$

$$\therefore \quad x = 2, \quad y = 3, \quad z = 5 \quad \text{Answer}$$

21. Solve for x and y if $\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} + 2\begin{bmatrix} 2x \\ 3y \end{bmatrix} = 3\begin{bmatrix} 7 \\ -3 \end{bmatrix}$. (ISC 2012)

Solution :

$$\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} + 2\begin{bmatrix} 2x \\ 3y \end{bmatrix} = 3\begin{bmatrix} 7 \\ -3 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x^2 \\ y^2 \end{bmatrix} + \begin{bmatrix} 4x \\ 6y \end{bmatrix} = \begin{bmatrix} 21 \\ -9 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x^2 + 4x \\ y^2 + 6y \end{bmatrix} = \begin{bmatrix} 21 \\ -9 \end{bmatrix}$$

$$\Rightarrow \quad x^2 + 4x = 21 \quad \text{and} \quad y^2 + 6y = -9$$

$$\Rightarrow \quad x^2 + 4x - 21 = 0 \quad \text{and} \quad y^2 + 6y + 9 = 0$$

$$\Rightarrow \quad x^2 + 7x - 3x - 21 = 0 \quad \text{and} \quad y^2 + 3y + 3y + 9 = 0$$

$$\Rightarrow \quad x(x+7) - 3(x+7) = 0 \quad \text{and} \quad y(y+3) + 3(y+3) = 0$$

$$\Rightarrow \quad (x+7)(x-3) = 0 \quad \text{and} \quad (y+3)(y+3) = 0$$

$\Rightarrow \quad x = -7,\ 3$ Answer and $y = -3, -3$ Answer

22. Find the product of the matrices A and B where (ISC 2012)

$$A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}.$$

Hence solve the following equations by matrix method:

$$x + y + 2z = 1$$
$$3x + 2y + z = 7$$
$$2x + y + 3z = 2$$

Solution : $AB = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} -5+3+6 & -5+2+3 & -10+1+9 \\ 7+3-10 & 7+2-5 & 14+1-15 \\ 1-3+2 & 1-2+1 & 2-1+3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$= 4I$ Answer

$$AB = 4I$$

$$\therefore \quad A(BB^{-1}) = 4\ (IB^{-1})$$

$$\Rightarrow \quad AI = 4\,B^{-1} \qquad (\because BB^{-1} = I \text{ and } IB^{-1} = B^{-1})$$

$$\Rightarrow \quad A = 4\,B^{-1} \qquad (\because AI = A)$$

$$\Rightarrow \quad B^{-1} = \frac{1}{4}A = \frac{1}{4}\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$$

Let $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $C = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$

The given equation may be written as

$$BX = C$$

$$\Rightarrow \quad X = B^{-1}C$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -5+7+6 \\ 7+7-10 \\ 1-7+2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ -4 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$$\Rightarrow \quad x = 2, \ y = 1, \ z = -1 \quad \text{Answer}$$

23. If $(B-2I)(B-3I) = 0$ where $B = \begin{bmatrix} 4 & 2 \\ -1 & y \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find the value of y.

(ISC 2013)

Solution : $B - 2I = \begin{bmatrix} 4 & 2 \\ -1 & y \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ -1 & y \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -1 & y-2 \end{bmatrix}$

and $B - 3I = \begin{bmatrix} 4 & 2 \\ -1 & y \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ -1 & y \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -1 & y-3 \end{bmatrix}$

$$\therefore \quad (B-2I)(B-3I) = \begin{bmatrix} 2 & 2 \\ -1 & y-2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & y-3 \end{bmatrix}$$

$$= \begin{bmatrix} 2-2 & 4+2y-6 \\ -1-y+2 & -2+y^2-5y+6 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2y-2 \\ 1-y & y^2-5y+4 \end{bmatrix}$$

Given $(B-2I)(B-3I) = 0$

$$\therefore \quad \begin{bmatrix} 0 & 2y-2 \\ 1-y & y^2-5y+4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\therefore \quad 1 - y = 0$$

$$\Rightarrow \quad y = 1 \quad \text{Answer}$$

24. Find A^{-1}, where $A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & -2 \end{bmatrix}$. (ISC 2013)

Hence solve the following system of linear equations:

$$4x + 2y + 3z = 2$$
$$x + y + z = 1$$
$$3x + y - 2z = 5$$

Solution : Given $A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 4 & -2 & -1 \\ 1 & 0 & 0 \\ 3 & -2 & -5 \end{bmatrix}$

$$= -1\begin{vmatrix} -2 & -1 \\ -2 & -5 \end{vmatrix} = -8 \neq 0$$

$$adj.A = \begin{bmatrix} +\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix} & +\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} \\ -\begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} & +\begin{vmatrix} 4 & 3 \\ 3 & -2 \end{vmatrix} & -\begin{vmatrix} 4 & 2 \\ 3 & 1 \end{vmatrix} \\ +\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 4 & 3 \\ 1 & 1 \end{vmatrix} & +\begin{vmatrix} 4 & 2 \\ 1 & 1 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} (-2-1) & -(-2-3) & (1-3) \\ -(-4-3) & (-8-9) & -(4-6) \\ (2-3) & -(4-3) & (4-2) \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & 5 & -2 \\ 7 & -17 & 2 \\ -1 & -1 & 2 \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & 7 & -1 \\ 5 & -17 & -1 \\ -2 & 2 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-8}\begin{bmatrix} -3 & 7 & -1 \\ 5 & -17 & -1 \\ -2 & 2 & 2 \end{bmatrix}$$ Answer

Let $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$

Now $AX = B$

$\Rightarrow \quad X = A^{-1}B$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-8}\begin{bmatrix} -3 & 7 & -1 \\ 5 & -17 & -1 \\ -2 & 2 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-8}\begin{bmatrix} -6+7-5 \\ 10-17-5 \\ -4+2+10 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-8}\begin{bmatrix} -4 \\ -12 \\ 8 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1/2 \\ 3/2 \\ -1 \end{bmatrix}$$

$\therefore \quad x = 1/2, \; y = 3/2, \; z = -1$ Answer.

25. If $B = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find the values of x and y such that $B^2 + xI_2 = yB$. (ISC 2014)

Solution : Given $B^2 + xI_2 = yB$

$$\Rightarrow \quad \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} + x\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = y\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 9+7 & 3+5 \\ 21+35 & 7+25 \end{bmatrix} + \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 16+x & 8 \\ 56 & 32+x \end{bmatrix} = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$$

$\therefore \quad 56 = 7y \implies y = 8$ Answer

$16 + x = 3y \implies 16 + x = 24 \implies x = 8$ Answer

26. Using matrix method , solve the following system of equation : (ISC 2014)

$x - 2y = 10, \quad 2x + y + 3z = 8, \quad -2y + z = 7$

Solution : The given system of equation is

$$x - 2y = 10$$
$$2x + y + 3z = 8$$
$$-2y + z = 7$$

Let $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}, \; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \; B = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & -2 & 0 \\ 2 & 3 & 1 \\ 0 & -2 & 1 \end{vmatrix} = 1\begin{vmatrix} 1 & 3 \\ -2 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & 3 \\ 0 & 1 \end{vmatrix} = 7 + 4 = 11 \neq 0$$

$$adj.A = \begin{bmatrix} +\begin{vmatrix} 1 & 3 \\ -2 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 3 \\ 0 & 1 \end{vmatrix} & +\begin{vmatrix} 2 & 1 \\ 0 & -2 \end{vmatrix} \\ -\begin{vmatrix} -2 & 0 \\ -2 & 1 \end{vmatrix} & +\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & -2 \\ 0 & -2 \end{vmatrix} \\ +\begin{vmatrix} -2 & 0 \\ 1 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix} & +\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} 7 & -2 & -4 \\ 2 & 1 & 2 \\ -6 & -3 & 5 \end{bmatrix}^T$$

$$= \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}\, adj\ A$$

$$= \frac{1}{11}\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$$

Now $AX = B$

$$\Longrightarrow \quad X = A^{-1}B$$

$$\Longrightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$\Longrightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 70 + 16 - 42 \\ -20 + 8 - 21 \\ -40 + 16 + 35 \end{bmatrix}$$

$$\Longrightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 44 \\ -33 \\ 11 \end{bmatrix}$$

$$\Longrightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$$

$$\Longrightarrow \quad x = 4, \ \ y = -3, \ \ z = 1$$ Answer

27. Find the value of k if $M = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $M^2 - kM - I_2 = 0$. (ISC 2015)

Solution : Given $M^2 - kM - I_2 = 0$

$$\Longrightarrow \quad \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} - k\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\Longrightarrow \quad \begin{bmatrix} 1+4 & 2+6 \\ 2+6 & 4+9 \end{bmatrix} - \begin{bmatrix} k & 2k \\ 2k & 3k \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\Longrightarrow \quad \begin{bmatrix} 5 & 8 \\ 8 & 13 \end{bmatrix} - \begin{bmatrix} k & 2k \\ 2k & 3k \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\Longrightarrow \quad \begin{bmatrix} 5-k-1 & 8-2k \\ 8-2k & 13-3k-1 \end{bmatrix} = 0$$

$$\Longrightarrow \quad \begin{bmatrix} 4-k & 8-2k \\ 8-2k & 12-3k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\therefore \quad 4 - k = 0 \quad \Longrightarrow k = 4$$ Answer

28. Given two matrices A and B (ISC 2015)

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 4 & 1 \\ 1 & -3 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 11 & -5 & -14 \\ -1 & -1 & 2 \\ -7 & 1 & 6 \end{bmatrix},$$

Find AB and use this result to solve the following system of equations :

$x - 2y + 3z = 6, \quad x + 4y + z = 12, \quad x - 3y + 2z = 1.$

Solution : $AB = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 4 & 1 \\ 1 & -3 & 2 \end{bmatrix} \begin{bmatrix} 11 & -5 & -14 \\ -1 & -1 & 2 \\ -7 & 1 & 6 \end{bmatrix}$

$= \begin{bmatrix} 11 + 2 - 21 & -5 + 2 + 3 & -14 - 4 + 18 \\ 11 - 4 - 7 & -5 - 4 + 1 & -14 + 8 + 6 \\ 11 + 3 - 14 & -5 + 3 + 2 & -14 - 6 + 12 \end{bmatrix}$

$= \begin{bmatrix} -8 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & -8 \end{bmatrix}$

$= -8 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore \quad AB = -8\, I_3$ Answer ------- (1)

Pre-multiplying equation (1) by A^{-1}

$(A^{-1}A)B = -8\, A^{-1} I_3$

$\therefore \quad I_3 B = -8A^{-1} \qquad [\because \ A^{-1}A = I_3, \quad A^{-1}I_3 = A^{-1}]$

$\therefore \quad A^{-1} = \frac{1}{-8} B \qquad [\because \ I_3 B = B\ ]$

$\therefore \quad A^{-1} = -\frac{1}{8} \begin{bmatrix} 11 & -5 & -14 \\ -1 & -1 & 2 \\ -7 & 1 & 6 \end{bmatrix}$

Let $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad C = \begin{bmatrix} 6 \\ 12 \\ 1 \end{bmatrix}$

Now $AX = C$

$\Rightarrow \quad X = A^{-1}C$

$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{8} \begin{bmatrix} 11 & -5 & -14 \\ -1 & -1 & 2 \\ -7 & 1 & 6 \end{bmatrix} \begin{bmatrix} 6 \\ 12 \\ 1 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{8} \begin{bmatrix} 66 - 60 - 14 \\ -6 - 12 + 2 \\ -42 + 12 + 6 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{8} \begin{bmatrix} -8 \\ -16 \\ -24 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$\therefore \quad x = 1, \quad y = 2, \quad z = 3$ Answer

29. Find the matrix X for which : (ISC 2016)

$\begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$

Solution : Let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\therefore \quad \begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} 5a + 4c & 5b + 4d \\ a + c & b + d \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$

$5a + 4c = 1$ ---------- (1)

$a + c = 1$ ---------- (2)

$5b + 4d = -2$ ---------- (3)

$$b + d = 3 \text{ ---------- (4)}$$

Solving (1) and (2), $a = -3,\ c = 4$

Solving (3) and (4), $b = -14,\ d = 17$

$$\therefore \quad X = \begin{bmatrix} -3 & -14 \\ 4 & 17 \end{bmatrix}$$ Answer

30. Solve the following system of linear equations using matrix method : (ISC 2016)

$3x + y + z = 1,\quad 2x + 2z = 0,\quad 5x + y + 2z = 2$

Solution : Let $A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 0 & 2 \\ 5 & 1 & 2 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix},\quad B = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$

$$\therefore \quad |A| = \begin{vmatrix} 3 & 1 & 1 \\ 2 & 0 & 2 \\ 5 & 1 & 2 \end{vmatrix}$$

$$= 3\begin{vmatrix} 0 & 2 \\ 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 2 & 2 \\ 5 & 2 \end{vmatrix} + 1\begin{vmatrix} 2 & 0 \\ 5 & 1 \end{vmatrix}$$

$$= -6 + 6 + 2 = 2 \neq 0$$

$$adj\ A = \begin{bmatrix} +\begin{vmatrix} 0 & 2 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & 2 \\ 5 & 2 \end{vmatrix} & +\begin{vmatrix} 2 & 0 \\ 5 & 1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} & +\begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} & -\begin{vmatrix} 3 & 1 \\ 5 & 1 \end{vmatrix} \\ +\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} & -\begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} & +\begin{vmatrix} 3 & 1 \\ 2 & 0 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} -2 & 6 & 2 \\ -1 & 1 & 2 \\ 2 & -4 & -2 \end{bmatrix}^T$$

$$= \begin{bmatrix} -2 & -1 & 2 \\ 6 & 1 & -4 \\ 2 & 2 & -2 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{2}\begin{bmatrix} -2 & -1 & 2 \\ 6 & 1 & -4 \\ 2 & 2 & -2 \end{bmatrix}$$

Now $AX = B$

$$\Rightarrow \quad X = A^{-1}B$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -2 & -1 & 2 \\ 6 & 1 & -4 \\ 2 & 2 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -2 + 4 \\ 6 - 8 \\ 2 - 4 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 \\ -2 \\ -2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\Rightarrow \quad x = 1,\quad y = -1,\quad z = -1$$ Answer

31. If the matrix $\begin{pmatrix} 6 & -x^2 \\ 2x - 15 & 10 \end{pmatrix}$ is symmetric, find the value of x. (ISC 2017)

Solution : As $\begin{pmatrix} 6 & -x^2 \\ 2x - 15 & 10 \end{pmatrix}$ is a symmetric matrix,

$$-x^2 = 2x - 15$$

$$\Rightarrow \quad x^2 + 2x - 15 = 0$$

$$\Rightarrow \quad (x+5)(x-3) = 0$$

$$\Rightarrow \quad x = -5,\ 3$$ Answer

32. Given $A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{pmatrix}$, find AB. (ISC 2017)

Using this result, solve the following system of equations.

$x - y = 3,\ \ 2x + 3y + 4z = 17$ and $y + 2z = 7$.

Solution : $$AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2+4 & 2-2 & -4+4 \\ 4-12+8 & 4+6-4 & -8-12+20 \\ -4+4 & 2-2 & -4+10 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$\therefore \quad AB = 6I$ Answer

$\therefore \quad B = 6\, I\, A^{-1}$

$\therefore \quad 6\, A^{-1} = B \qquad [\because \ IA^{-1} = A^{-1}\,]$

$\therefore \quad A^{-1} = \frac{1}{6} B$

$$= \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

Let $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix},\ B = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

Now $AX = B$

$\Rightarrow \quad X = A^{-1} B$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 6+34-28 \\ -12+34-28 \\ 6-17+35 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$\Rightarrow \quad x = 2, \quad y = -1, \quad z = 4$ Answer

33. If $A = \begin{pmatrix} 5 & a \\ b & 0 \end{pmatrix}$ and A is a symmetric matrix, show that $a = b$. (ISC 2018)

Solution : Given $A = \begin{pmatrix} 5 & a \\ b & 0 \end{pmatrix}$.

As A is a symmetric matrix, $a_{ij} = a_{ji}$.

$\therefore \quad a_{12} = a_{21}$

$\therefore \quad a = b$ Proved

34. Using matrices, solve the following system of equations : (ISC 2018)

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

Solution : Given $2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$, $x + y - 2z = -3$

Let $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

$\therefore \quad |A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$

$= 2\begin{vmatrix} 2 & -4 \\ 1 & -2 \end{vmatrix} + 3\begin{vmatrix} 3 & -4 \\ 1 & -2 \end{vmatrix} + 5\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix}$

$= 2(-4 + 4) + 3(-6 + 4) + 5(3 - 2)$

$= -6 + 5 = -1 \neq 0$

$$adj.A = \begin{bmatrix} +\begin{vmatrix} 2 & -4 \\ 1 & -2 \end{vmatrix} & -\begin{vmatrix} 3 & -4 \\ 1 & -2 \end{vmatrix} & +\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} \\ -\begin{vmatrix} -3 & 5 \\ 1 & -2 \end{vmatrix} & +\begin{vmatrix} 2 & 5 \\ 1 & -2 \end{vmatrix} & -\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} \\ +\begin{vmatrix} -3 & 5 \\ 2 & -4 \end{vmatrix} & -\begin{vmatrix} 2 & 5 \\ 3 & -4 \end{vmatrix} & +\begin{vmatrix} 2 & -3 \\ 3 & 2 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} (-4 + 4) & -(-6 + 4) & (3 - 2) \\ -(6 - 5) & (-4 - 5) & -(2 + 3) \\ (12 - 10) & -(-8 - 15) & (4 + 9) \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$\therefore \quad A^{-1} = -1\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

Now $AX = B$

$\Rightarrow \quad X = A^{-1}B$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\begin{bmatrix} 5 - 6 \\ 22 + 45 - 69 \\ 11 + 25 - 39 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\Rightarrow \quad x = 1, \quad y = 2, \quad z = 3$ Answer

35. Using elementary transformation, find the inverse of the matrix : (ISC 2018)

$$\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix}$$

Solution : Let $A = \begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix}$

We have

$$\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}.A$$

$$\Rightarrow \quad \begin{bmatrix}1 & 3 & 5\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix} = \begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}.A \qquad [R_1 \to R_1 + R_2]$$

$$\Rightarrow \quad \begin{bmatrix}1 & 3 & 5\\ 0 & -1 & -2\\ 0 & -8 & -14\end{bmatrix} = \begin{bmatrix}1 & 1 & 0\\ -1 & 0 & 0\\ -3 & -3 & 1\end{bmatrix}.A \qquad \begin{matrix}R_2 \to R_2 - R_1\\ R_3 \to R_3 - 3R_1\end{matrix}$$

$$\Rightarrow \quad \begin{bmatrix}1 & 3 & 5\\ 0 & 1 & 2\\ 0 & -8 & -14\end{bmatrix} = \begin{bmatrix}1 & 1 & 0\\ 1 & 0 & 0\\ -3 & -3 & 1\end{bmatrix}.A \qquad R_2 \to (-1)R_2$$

$$\Rightarrow \quad \begin{bmatrix}1 & 0 & -1\\ 0 & 1 & 2\\ 0 & 0 & 2\end{bmatrix} = \begin{bmatrix}-2 & 1 & 0\\ 1 & 0 & 0\\ 5 & -3 & 1\end{bmatrix}.A \qquad \begin{matrix}R_1 \to R_1 - 3R_2\\ R_3 \to R_3 + 8R_2\end{matrix}$$

$$\Rightarrow \quad \begin{bmatrix}1 & 0 & -1\\ 0 & 1 & 2\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}-2 & 1 & 0\\ 1 & 0 & 0\\ 5/2 & -3/2 & 1/2\end{bmatrix}.A \qquad R_3 \to \frac{1}{2}R_3$$

$$\Rightarrow \quad \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1/2 & -1/2 & 1/2\\ -4 & 3 & -1\\ 5/2 & -3/2 & 1/2\end{bmatrix}.A \qquad \begin{matrix}R_1 \to R_1 + R_3\\ R_2 \to R_2 - 2R_3\end{matrix}$$

Hence $A^{-1} = \begin{bmatrix}1/2 & -1/2 & 1/2\\ -4 & 3 & -1\\ 5/2 & -3/2 & 1/2\end{bmatrix}$ Answer

36. Show that $(A + A')$ is symmetric matrix, if $A = \begin{pmatrix}2 & 3\\ 3 & 5\end{pmatrix}$ (ISC 2019)

Solution : Given $A = \begin{pmatrix}2 & 4\\ 3 & 5\end{pmatrix}$

$$\therefore \quad A' = \begin{pmatrix}2 & 3\\ 4 & 5\end{pmatrix}$$

$$\therefore \quad A + A' = \begin{pmatrix}2 & 4\\ 3 & 5\end{pmatrix} + \begin{pmatrix}2 & 3\\ 4 & 5\end{pmatrix}$$

$$= \begin{pmatrix}4 & 7\\ 7 & 10\end{pmatrix}$$

As $a_{12} = a_{21} = 7$, $A + A'$ is a symmetric matrix.

37. Solve the following system of linear equations using matrix method : (ISC 2019)

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 9$$

$$\frac{2}{x} + \frac{5}{y} + \frac{7}{z} = 52$$

$$\frac{2}{x} + \frac{1}{y} - \frac{1}{z} = 0$$

Solution : Let $\frac{1}{x} = a, \quad \frac{1}{y} = b, \quad \frac{1}{z} = c$

The given equation may be written as

$$a+b+c=9$$
$$2a+5b+7c=52$$
$$2a+b-c=0$$

Let $A=\begin{bmatrix}1 & 1 & 1\\ 2 & 5 & 7\\ 2 & 1 & -1\end{bmatrix}$, $X=\begin{bmatrix}a\\ b\\ c\end{bmatrix}$, $B=\begin{bmatrix}9\\ 52\\ 0\end{bmatrix}$

$$\therefore \quad |A|=\begin{vmatrix}1 & 1 & 1\\ 2 & 5 & 7\\ 2 & 1 & -1\end{vmatrix}=\begin{vmatrix}1 & 0 & 0\\ 2 & 3 & 5\\ 2 & -1 & -3\end{vmatrix}$$

$$=1\begin{vmatrix}3 & 5\\ -1 & -3\end{vmatrix}=-9+5=-4\neq 0$$

$$adj\ A=\begin{bmatrix}+\begin{vmatrix}5 & 7\\ 1 & -1\end{vmatrix} & -\begin{vmatrix}2 & 7\\ 2 & -1\end{vmatrix} & +\begin{vmatrix}2 & 5\\ 2 & 1\end{vmatrix}\\ -\begin{vmatrix}1 & 1\\ 1 & -1\end{vmatrix} & +\begin{vmatrix}1 & 1\\ 2 & -1\end{vmatrix} & -\begin{vmatrix}1 & 1\\ 2 & 1\end{vmatrix}\\ +\begin{vmatrix}1 & 1\\ 5 & 7\end{vmatrix} & -\begin{vmatrix}1 & 1\\ 2 & 7\end{vmatrix} & +\begin{vmatrix}1 & 1\\ 2 & 5\end{vmatrix}\end{bmatrix}^T$$

$$=\begin{bmatrix}(-5-7) & -(-2-14) & (2-10)\\ -(-1-1) & (-1-2) & -(1-2)\\ (7-5) & -(7-2) & (5-2)\end{bmatrix}^T$$

$$=\begin{bmatrix}-12 & 16 & -8\\ 2 & -3 & 1\\ 2 & -5 & 3\end{bmatrix}^T$$

$$=\begin{bmatrix}-12 & 2 & 2\\ 16 & -3 & -5\\ -8 & 1 & 3\end{bmatrix}$$

$$\therefore \quad A^{-1}=\frac{1}{|A|}\,adj\ A$$

$$=\frac{1}{-4}\begin{bmatrix}-12 & 2 & 2\\ 16 & -3 & -5\\ -8 & 1 & 3\end{bmatrix}$$

Now $AX=B$

$$\Longrightarrow \quad X=A^{-1}B$$

$$\Longrightarrow \quad \begin{bmatrix}a\\ b\\ c\end{bmatrix}=-\frac{1}{4}\begin{bmatrix}-12 & 2 & 2\\ 16 & -3 & -5\\ -8 & 1 & 3\end{bmatrix}\begin{bmatrix}9\\ 52\\ 0\end{bmatrix}$$

$$\Longrightarrow \quad \begin{bmatrix}a\\ b\\ c\end{bmatrix}=-\frac{1}{4}\begin{bmatrix}-108+104\\ 144-156\\ -72+52\end{bmatrix}$$

$$\Longrightarrow \quad \begin{bmatrix}a\\ b\\ c\end{bmatrix}=-\frac{1}{4}\begin{bmatrix}-4\\ -12\\ -20\end{bmatrix}$$

$$\Longrightarrow \quad \begin{bmatrix}1/x\\ 1/y\\ 1/z\end{bmatrix}=\begin{bmatrix}1\\ 3\\ 5\end{bmatrix}$$

$$\therefore \quad x=1, \quad y=1/3, \quad z=1/5$$ Answer

38. If $\begin{pmatrix}2 & 3\\ 5 & 7\end{pmatrix}\begin{pmatrix}1 & -3\\ -2 & 4\end{pmatrix}=\begin{pmatrix}-4 & 6\\ -9 & x\end{pmatrix}$, find x. (ISC 2020)

Solution : Given $\begin{pmatrix}2 & 3\\ 5 & 7\end{pmatrix}\begin{pmatrix}1 & -3\\ -2 & 4\end{pmatrix}=\begin{pmatrix}-4 & 6\\ -9 & x\end{pmatrix}$

$$\Longrightarrow \quad \begin{pmatrix}2-6 & -6+12\\ 5-14 & -15+28\end{pmatrix}=\begin{pmatrix}-4 & 6\\ -9 & x\end{pmatrix}$$

$$\Rightarrow \quad \begin{pmatrix} -4 & 6 \\ -9 & 13 \end{pmatrix} = \begin{pmatrix} -4 & 6 \\ -9 & x \end{pmatrix}$$

$$\Rightarrow \quad x = 13 \quad \text{Answer}$$

39. Solve the following system of linear equations using matrices : (ISC 2020)

$$x - 2y = 10, \quad 2x - y - z = 8, \quad -2y + z = 7$$

Solution : The given equations are

$$x - 2y = 10$$
$$2x - y - z = 8$$
$$-2y + z = 7$$

Let $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

$$\therefore \quad |A| = \begin{vmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{vmatrix} = 1\begin{vmatrix} -1 & -1 \\ -2 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix}$$

$$= -1 - 2 + 4 = 1 \neq 0$$

$$adj\ A = \begin{bmatrix} +\begin{vmatrix} -1 & -1 \\ -2 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix} & +\begin{vmatrix} 2 & -1 \\ 0 & -2 \end{vmatrix} \\ -\begin{vmatrix} -2 & 0 \\ -2 & 1 \end{vmatrix} & +\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & -2 \\ 0 & -2 \end{vmatrix} \\ +\begin{vmatrix} -2 & 0 \\ -1 & -1 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 2 & -1 \end{vmatrix} & +\begin{vmatrix} 1 & -2 \\ 2 & -1 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}\ adj\ A$$

$$\therefore \quad A^{-1} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}$$

Now $AX = B$

$$\Rightarrow \quad X = A^{-1}\ B$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -30 + 16 + 14 \\ -20 + 8 + 7 \\ -40 + 16 + 21 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

$$\Rightarrow \quad x = 0, \quad y = -5, \quad z = -3 \quad \text{Answer}$$

Matrices

(Solutions of Board Questions)

Semester 1 - 2021

Select and write the correct option:

40. If $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$, then the matrix A^2 is equal to:

(a) $A^2 = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 16 \end{pmatrix}$

(b) $A^2 = \begin{pmatrix} 0 & 0 & 4 \\ 0 & 9 & 0 \\ 16 & 0 & 0 \end{pmatrix}$

(c) $A^2 = \begin{pmatrix} 4 & 0 & 16 \\ 0 & 0 & 0 \\ 0 & 9 & 0 \end{pmatrix}$

(d) $A^2 = \begin{pmatrix} 0 & 0 & 0 \\ 4 & 9 & 16 \\ 0 & 0 & 0 \end{pmatrix}$

Answer: _____(a)_____

Solution: $A^2 = A.A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}\begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix} = \begin{pmatrix} 2\times 2 & 0 & 0 \\ 0 & 3\times 3 & 0 \\ 0 & 0 & 4\times 4 \end{pmatrix} = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 16 \end{pmatrix}$

41. If $2\begin{pmatrix} a & 9 \\ 6 & d \end{pmatrix} + 3\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} = 3\begin{pmatrix} 3 & 5 \\ 4 & 6 \end{pmatrix}$, then the values of a and d respectively are:

(a) 3, 6

(b) 3, 5

(c) 3, 9

(d) 3, 7

Answer: _____(a)_____

Solution: $2\begin{pmatrix} a & 9 \\ 6 & d \end{pmatrix} + 3\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} = 3\begin{pmatrix} 3 & 5 \\ 4 & 6 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 2a & 18 \\ 12 & 2d \end{pmatrix} + \begin{pmatrix} 3 & -3 \\ 0 & 6 \end{pmatrix} = \begin{pmatrix} 9 & 15 \\ 12 & 18 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 2a+3 & 15 \\ 12 & 2d+6 \end{pmatrix} = \begin{pmatrix} 9 & 15 \\ 12 & 18 \end{pmatrix}$

$\therefore \quad 2a + 3 = 9 \Rightarrow a = 3$ and $2d + 6 = 18 \Rightarrow d = 6$

42. If the matrix $A = \begin{pmatrix} 1 & x & -1 \\ -1 & 3 & 2 \\ 2 & 1 & 1 \end{pmatrix}$ is singular, then the value of $'x'$ is

(a) $\frac{8}{5}$

(b) $\frac{-8}{5}$

(c) $\frac{5}{8}$

(d) 1

Answer: (b)

Solution: $|A| = 0$ (as A is a singular matrix)

$$\Rightarrow \begin{vmatrix} 1 & x & -1 \\ -1 & 3 & 2 \\ 2 & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & x & 0 \\ -1 & 3 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 0 \quad \text{(applying } C_3 \to C_3 + C_1\text{)}$$

$$\Rightarrow 1(9-1) - x(-3-2) = 0 \quad \Rightarrow \quad 8 + 5x = 0 \quad \Rightarrow \quad x = \frac{-8}{5}$$

43. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, then the matrix $A + A^T$ is

(a) Symmetric matrix

(b) Skew-symmetric matrix

(c) Diagonal matrix

(d) Identity matrix

Answer: (a)

Solution: $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad \therefore A^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$

$$\therefore \quad A + A^T = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ 5 & 8 \end{pmatrix}$$

44. If $A = \begin{pmatrix} -1 & 1 \\ 2 & 3 \end{pmatrix}$, then $A\ (adj\ A)$ is equal to:

(a) $\begin{pmatrix} 4 & 0 \\ 10 & 24 \end{pmatrix}$

(b) $\begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix}$

(c) $\begin{pmatrix} 4 & 5 \\ 5 & 4 \end{pmatrix}$

(d) $\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$

Answer: (b)

Solution: $(adj\ A) = \begin{pmatrix} 3 & -1 \\ -2 & -1 \end{pmatrix}$

$$\therefore \quad A\ (adj\ A) = \begin{pmatrix} -1 & 1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 3 & -1 \\ -2 & -1 \end{pmatrix} = \begin{pmatrix} -3-2 & 1-1 \\ 6-6 & -2-3 \end{pmatrix} = \begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix}$$

45. A school wants to award its students for their achievement in Sports, Music, and Literature with a total cash prize of ₹6000.
Three times the prize money for Literature added to the prize money given for sports is equal to ₹11000.
The prize money given for sports and Literature together is equal to two times of the prize money given for music.
If x, y, and z represent the prize money given for Sports, Music, and Literature respectively, then

(i) The set of linear equations representing the above information will be:

(a) $x + y + z = 6000,\ x + 3z = 11000$ and $x - 2y + z = 0$
(b) $x + y + z = 6000,\ x + 3z = 11000$ and $x + y + 2z = 0$
(c) $x + y + z = 6000,\ 3x + z = 11000$ and $x + y - 2z = 0$
(d) $x + y + z = 6000,\ x + 3z = 11000$ and $2x + 2y - z = 0$

Answer: (a)

(ii) Consider $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{pmatrix}$

Then the value of $|A|$ is

(a) 3
(b) 6
(c) 4
(d) -1

Answer: (b)

Solution:

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & -1 & 2 \\ 1 & -3 & 0 \end{vmatrix} = 1(0 + 6) = 6$$

(iii) The $adj\ A = \begin{pmatrix} 6 & p & 3 \\ q & 0 & r \\ -2 & 3 & -1 \end{pmatrix}$, the values of p, q and r respectively will be:

(a) $3,\ 2, -2$
(b) $3, -2, -2$
(c) $-3,\ 2,\ -2$
(d) $3,\ 2,\ 2$

Answer: (c)

Solution:

$$adj\ A = \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$$

$$p = A_{21} = -\begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix} = -3$$

$$q = A_{12} = -\begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} = 2$$

$$r = A_{32} = -\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = -2$$

(iv) Using $|A|$ and $adj\ A$, calculate the prize money for sports (x).

(a) $x = 1500$
(b) $x = 500$
(c) $x = 1000$
(d) $x = 2000$

Answer: (b)

Solution:

$$X = A^{-1} B$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$$

$$\therefore \quad x = \frac{1}{6}\{6 \times 6000 + (-3)11000 + 3 \times 0\} = \frac{1}{6} \times 3000 = 500$$